Please check the examination details below before entering your candidate information

Candidate surname	Other names
MODEL SOLUTIONS	

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Time 1 hour 30 minutes

Paper reference **9FM0/4C**

Further Mathematics

Advanced

PAPER 4C: Further Mechanics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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1. Three particles of masses $2m$, $3m$ and km are placed at the points with coordinates $(3a, 2a)$, $(a, -4a)$ and $(-3a, 4a)$ respectively.

The centre of mass of the three particles lies at the point with coordinates $(\bar{x}, \bar{y})$.

(a) (i) Find $\bar{x}$ in terms of a and k

(ii) Find $\bar{y}$ in terms of a and k

(4)

Given that the distance of the centre of mass of the three particles from the point $(0, 0)$ is $\frac{1}{3}a$

(b) find the possible values of k

(2)

1) i) Take moments about the y axis. 1

$\bar{x}(2m + 3m + km) = 2m(3a) + 3m(a) + km(-3a)$

$\Rightarrow (5+k)m\bar{x} = (9 - 3k)ma$

$\Rightarrow \bar{x} = \frac{(9-3k)a}{5+k}$ 1

ii) Take moments about the x-axis. 1

$\bar{y}(2m + 3m + km) = 2m(2a) + 3m(-4a) + km(4a)$

$\Rightarrow (5+k)m\bar{y} = (4k - 8)ma$

$\Rightarrow \bar{y} = \frac{(4k-8)a}{5+k}$ 1

b) Recall that the distance from the origin is $\sqrt{\bar{x}^2 + \bar{y}^2}$

$\frac{1}{3}a = \sqrt{\left(\frac{(9-3k)a}{5+k}\right)^2 + \left(\frac{(4k-8)a}{5+k}\right)^2}$

We can immediately cancel out the a.

Question 1 continued

$\Rightarrow \frac{1}{9} = \left(\frac{9-3k}{5+k}\right)^2 + \left(\frac{4k-8}{5+k}\right)^2$ (1)

$\Rightarrow (5+k)^2 = 9(9-3k)^2 + 9(4k-8)^2$

$\Rightarrow k^2 + 10k + 25 = 9(9k^2 - 54k + 81) + 9(16k^2 - 64k + 64)$

$\Rightarrow 224k^2 - 1072k + 1280 = 0$

$\Rightarrow k = 5/2$ or $k = 16/7$ using a calculator. (1)

(Total for Question 1 is 6 marks)

2. A cyclist and her cycle have a combined mass of 60 kg. The cyclist is moving along a straight horizontal road and is working at a constant rate of 200 W.

When she has travelled a distance x metres, her speed is v m s^{-1} and the magnitude of the resistance to motion is $3v^2$ N.

(a) Show that $\dfrac{dv}{dx} = \dfrac{200 - 3v^3}{60v^2}$

(4)

The distance travelled by the cyclist as her speed increases from 2 m s^{-1} to 4 m s^{-1} is D metres.

(b) Find the exact value of D

(3)

a) Recall that Force $= \dfrac{\text{Power}}{\text{velocity}}$ and $F = ma$

$F = ma$

$\Rightarrow \quad F = mv \cdot \dfrac{dv}{dx}$ (1)

$\Rightarrow \quad \dfrac{200}{v} - 3v^2 = 60v \cdot \dfrac{dv}{dx}$ (1) (1) $\quad F = \dfrac{200}{v}$

$\Rightarrow \quad \dfrac{200 - 3v^3}{v} = 60v \cdot \dfrac{dv}{dx}$

$\Rightarrow \quad \dfrac{200 - 3v^3}{60v^2} = \dfrac{dv}{dx}$ (1)

b) This is a separable ODE.

$\int dx = \int \dfrac{60v^2}{200 - 3v^3} \, dv$ (1)

We can either use a substitution, or see that this is reverse chain rule as $\frac{d}{dv}(v^3) = v^2$, so we will have a multiple of a natural log.

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Question 2 continued

$\Rightarrow \quad x = -\frac{60}{9}\ln(200-3v^3) + c$

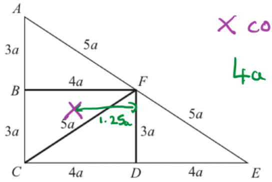
When $v=2$, $x = \frac{-60}{9}\ln(176) + c$

When $v=4$, $x = -\frac{60}{9}\ln(8) + c$

$D = -\frac{60}{9}\ln(8) + c - \left[-\frac{60}{9}\ln(176) + c\right]$ 1

$\Rightarrow D = \frac{60}{9}\ln\left(\frac{176}{8}\right) = \frac{60}{9}\ln(22)$ 1

3.



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$4a - \bar{x} = 1.25$

Figure 1

Nine uniform rods are joined together to form the rigid framework $ABCDEFA$, with $AB = BC = DF = 3a$, $BF = CD = DE = 4a$ and $AF = FE = CF = 5a$, as shown in Figure 1. All nine rods lie in the same plane.

The mass per unit length of each of the rods BF, CF and DF is twice the mass per unit length of each of the other six rods.

(a) Find the distance of the centre of mass of the framework from AC

(4)

The mass of the framework is M. A particle of mass kM is attached to the framework at E to form a loaded framework.

When the loaded framework is freely suspended from F, it hangs in equilibrium with CE horizontal.

(b) Find the exact value of k

(3)

a) We take moments about AC. 1

rod	CD	DE	EF	FA	AB	BC	BF	DF	CF
Mass ratio	4	4	5	5	3	3	8	6	10
From AC	2a	6a	6a	2a	0	0	2a	4a	2a

BF, CF and DF have double mass

$\bar{x} \times \text{Mass} = \Sigma(\text{Mass} \times \text{distance})$

LHS: $\bar{x}(4+4+5+5+3+3+8+6+10) = 48\bar{x}$ 1

Question 3 continued

RHS: $4(2a) + 4(6a) + 5(6a) + 5(2a) + 8(2a) + 6(4a) + 10(2a)$ 1

$= 132a$

$\Rightarrow 48\bar{x} = 132a$

$\Rightarrow \bar{x} = \frac{11}{4}a$ 1

b) Take moments about F. 1

$1.25a\,Mg = 4a\,(kMg)$ 1 Using the diagram

$\Rightarrow k = \frac{5}{16}$ 1

Turn over

4.

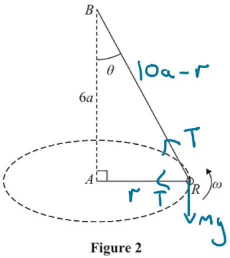


Figure 2

A small smooth ring R of mass m is threaded onto a light inextensible string. One end of the string is attached to a fixed point A and the other end of the string is attached to the fixed point B such that B is vertically above A and $AB = 6a$

The ring moves with constant angular speed ω in a horizontal circle with centre A. The string is taut and BR makes a constant angle θ with the downward vertical, as shown in Figure 2.

The ring is modelled as a particle.

Given that $\tan\theta = \frac{8}{15}$

(a) find, in terms of m and g, the magnitude of the tension in the string,

(3)

(b) find ω in terms of a and g

(5)

a) $\tan\theta = \frac{8}{15} \Rightarrow \sin\theta = \frac{8}{17}, \cos\theta = \frac{15}{17}$

Resolve Vertically 1

$T\cos\theta = mg$ 1

$\Rightarrow T\left(\frac{15}{7}\right) = mg \Rightarrow T = \frac{7}{15}mg$ 1

Question 4 continued

b) Recall that $F = ma$ and $a = \omega^2 r$

Resolve Horizontally

$T + T\sin\theta = mr\omega^2$ (1)

$r = 6a\tan\theta = \frac{16}{5}a$

$\Rightarrow \frac{17}{15}mg\left(1 + \frac{8}{17}\right) = m\omega^2\left(\frac{16}{5}a\right)$ (1) (1)

$\Rightarrow \frac{5}{3}g = \omega^2\left(\frac{16}{5}a\right)$

$\Rightarrow \frac{25}{3}g = 16a\omega^2$

$\Rightarrow \omega^2 = \frac{25g}{48a}$ (1)

$\Rightarrow \omega = \sqrt{\frac{25g}{48a}}$ (1)

• Qa

• Qb

5.

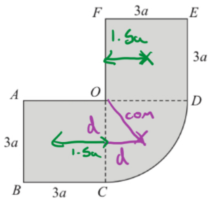


Figure 3

The uniform plane lamina shown in Figure 3 is formed from two squares, $ABCO$ and $ODEF$, and a sector ODC of a circle with centre O. Both squares have sides of length $3a$ and AO is perpendicular to OF. The radius of the sector is $3a$

[*In part* (a) *you may use, without proof, any of the centre of mass formulae given in the formulae booklet.*]

(a) Show that the distance of the centre of mass of the sector ODC from OC is $\frac{4a}{\pi}$ **(3)**

(b) Find the distance of the centre of mass of the lamina from FC **(4)**

The lamina is freely suspended from F and hangs in equilibrium with FC at an angle $\theta°$ to the downward vertical.

(c) Find the value of θ **(4)**

a) Using the formula booklet, for a sector of a circle with radius r, angle at centre 2α, the CoM is $\frac{2r\sin\alpha}{3\alpha}$ from the centre.

$r = 3a$, $\alpha = \frac{\pi}{4}$

So CoM $= \frac{2(3a)\sin(\pi/4)}{3(\pi/4)} = \frac{8a}{\pi\sqrt{2}}$ 1

P 7 2 1 1 4 A 0 1 6 3 2

Question 5 continued

We want the CoM from OC not O.

Using Pythagoras and the diagram,

$$d^2 + d^2 = \left(\frac{8a}{\pi\sqrt{2}}\right)^2 \quad 1$$

$$\Rightarrow \quad 2d^2 = \frac{64a^2}{2\pi^2}$$

$$\Rightarrow \quad d^2 = \frac{16a^2}{\pi^2}$$

$$\Rightarrow \quad d = \frac{4a}{\pi} \quad 1$$

b) $\bar{x} \times \text{Mass} = \Sigma \text{ distance} \times \text{mass}$

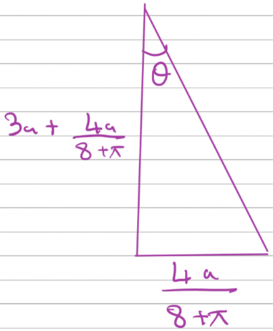
$$\bar{x}\left(9a^2 + 9a^2 + \frac{9}{4}\pi a^2\right) = 1.5a(9a^2) - 1.5a(9a^2) + \frac{4a}{\pi}\left(\frac{9}{4}\pi a^2\right) \quad 1 \quad 1$$

$$\Rightarrow \quad \bar{x}\left(18 + \frac{9}{4}\pi\right) = 9a \quad 1$$

$$\Rightarrow \quad \bar{x} = \frac{4a}{8+\pi} \quad 1$$

Question 5 continued

c)



1

$$3a + \frac{4a}{8+\pi} = \frac{28a + 3a\pi}{8+\pi}$$

$$\tan\theta = \frac{\frac{4a}{8+\pi}}{\frac{28a+3a\pi}{8+\pi}}$$ 1

$$\Rightarrow \tan\theta = \frac{4}{28+3\pi}$$

$$\Rightarrow \theta = 6.1$$ 1

6.

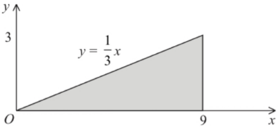


Figure 4

The shaded region shown in Figure 4 is bounded by the x-axis, the line with equation $x = 9$ and the line with equation $y = \frac{1}{3}x$. This shaded region is rotated through 360° about the x-axis to form a solid of revolution. This solid of revolution is used to model a solid right circular cone of height 9 cm and base radius 3 cm.

The cone is non-uniform and the mass per unit volume of the cone at the point (x, y, z) is λx kg cm^{-3}, where $0 \leqslant x \leqslant 9$ and λ is constant.

(a) Find the distance of the centre of mass of the cone from its vertex.

(6)

A toy is made by joining the circular plane face of the cone to the circular plane face of a uniform solid hemisphere of radius 3 cm, so that the centres of the two plane surfaces coincide.

The weight of the cone is W newtons and the weight of the hemisphere is kW newtons.

When the toy is placed on a smooth horizontal plane with any point of the curved surface of the hemisphere in contact with the plane, the toy will remain at rest.

(b) Find the value of k

(4)

a) Recall that Mass $= \pi \int \rho y^2 \, dx$

$$\text{Mass} = \pi \int_0^9 (\lambda x)\left(\frac{1}{3}x\right)^2 dx \quad (1)$$

$$= \lambda\pi \left[\frac{x^4}{36}\right]_0^9 = \frac{729}{4}\lambda\pi \quad (1)$$

Recall that $\bar{x} = \dfrac{\pi \int \rho x y^2 \, dx}{\text{Mass}}$

Question 6 continued

$$\bar{x} = \frac{\pi \int (\lambda x)(x)\left(\frac{1}{3}\right)x^2 \, dx}{\frac{729}{4}\lambda\pi} \quad 1$$

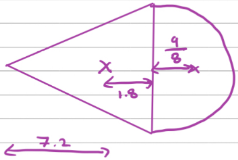
$$\Rightarrow \bar{x} = \frac{\lambda\pi \int_0^9 \frac{1}{9} x^4 \, dx}{\frac{729}{4}\lambda\pi} \quad 1$$

$$\Rightarrow \bar{x} = \frac{\left[\frac{x^5}{45}\right]_0^9}{\frac{729}{4}} \quad 1 \quad = 7.2\text{cm}. \quad 1$$

b) If the object remains at rest, then the centre of mass is at the centre of plane surface. 1

Using the formula booklet, the COM of a solid hemisphere with radius r is $3/8\, r$ from the centre.

COM is $\frac{9}{8}$ from the centre.



Question 6 continued

Take moments about the diameter of plane surface. 1

1 $1.8W = \frac{9}{8}kW \Rightarrow k = \frac{8}{5}$ 1

7.

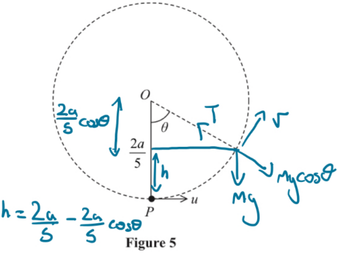


Figure 5

A package P of mass m is attached to one end of a string of length $\frac{2a}{5}$. The other end of the string is attached to a fixed point O. The package hangs at rest vertically below O with the string taut and is then projected horizontally with speed u, as shown in Figure 5.

When OP has turned through an angle θ and the string is still taut, the tension in the string is T

The package is modelled as a particle and the string as being light and inextensible.

(a) Show that $T = 3mg\cos\theta - 2mg + \dfrac{5mu^2}{2a}$
(6)

Given that P moves in a complete vertical circle with centre O

(b) find, in terms of a and g, the minimum possible value of u
(2)

Given that $u = 2\sqrt{ag}$

(c) find, in terms of g, the magnitude of the acceleration of P at the instant when OP is horizontal.
(3)

(d) Apart from including air resistance, suggest one way in which the model could be refined to make it more realistic.
(1)

a) Recall that $F = ma$ and $a = \frac{v^2}{r}$

Question 7 continued

Resolving forces, 1

$T - Mg\cos\theta = \frac{Mv^2}{r}$ 1

$\Rightarrow T - Mg\cos\theta = \frac{5Mv^2}{2a}$ (1) 1 $\quad r = \frac{2a}{5}$

Recall that $E_k = \frac{1}{2}mv^2$, $E_p = mgh$, and energy is conserved.

$\frac{1}{2}Mu^2 = \frac{1}{2}Mv^2 + Mgh$ 1 Using the diagram,

$\Rightarrow \frac{1}{2}Mu^2 = \frac{1}{2}Mv^2 + Mg\left(\frac{2a}{5} - \frac{2a}{5}\cos\theta\right)$

$\Rightarrow 5Mu^2 = 5Mv^2 + 4Mga(1-\cos\theta)$

$\Rightarrow 5u^2 - 4ga(1-\cos\theta) = 5v^2$ 1 Sub this into (1)

$\Rightarrow T - Mg\cos\theta = \frac{M}{2a}\left(5u^2 - 4ga(1-\cos\theta)\right)$

$\Rightarrow T - Mg\cos\theta = \frac{5Mu^2}{2a} - 2Mg + 2Mg\cos\theta$

$\Rightarrow T = 3Mg\cos\theta - 2Mg + \frac{5Mu^2}{2a}$ 1

Question 7 continued

b) T must always be greater than 0 in order for circular motion to continue. We find the minimum possible value of u by comparing with $\pi = \theta$.

$T \geq 0$ when $\theta = \pi$ implies that

$3mg\cos\pi - 2mg + \frac{5mu^2}{2a} \geq 0$ (1)

$\Rightarrow \quad -5g + \frac{5u^2}{2a} \geq 0$

$\Rightarrow \quad u^2 \geq 2ag$

$\Rightarrow \quad u \geq \sqrt{2ag}$ So the minimum $u = \sqrt{2ag}$ (1)

c) When OP is horizontal, $\theta = \frac{\pi}{2}$

when $\theta = \pi/2$, $u = 2\sqrt{ag}$, (1)

$T = 3mg\cos(\pi/2) - 2mg + \frac{5m(2\sqrt{ag})^2}{2a}$

$\Rightarrow T = -2mg + \frac{20mag}{2a}$

$\Rightarrow T = -8mg \Rightarrow a = -8g$

We know that the vertical acceleration is g downwards.

(1)

$a = \sqrt{(-8g)^2 + g^2} = \sqrt{65}\,g$ (1)

Question 7 continued

d) We have assumed that the string is inextensible, but it may be extensible. ①

(Total for Question 7 is 12 marks)

8. **Throughout this question, use $g = 10\,\text{m s}^{-2}$**

A light elastic string has natural length 1.25 m and modulus of elasticity 25 N.

A particle P of mass 0.5 kg is attached to one end of the string. The other end of the string is attached to a fixed point A. Particle P hangs freely in equilibrium with P vertically below A

The particle is then pulled vertically down to a point B and released from rest.

(a) Show that, while the string is taut, P moves with simple harmonic motion with period $\frac{\pi}{\sqrt{10}}$ seconds.

(6)

The maximum kinetic energy of P during the subsequent motion is 2.5 J.

(b) Show that $AB = 2\,\text{m}$

(3)

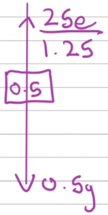
The particle returns to B for the first time T seconds after it was released from rest at B

(c) Find the value of T

(5)

a) Recall that $T = \frac{\lambda x}{l}$ and $F = ma = m\ddot{x}$

$T = \frac{25e}{1.25}$



$\frac{25e}{1.25} = 0.5g$

$\Rightarrow e = 0.25$ (1)

$T = m\ddot{x}$ (1)

$\Rightarrow \frac{25(0.25+x)}{1.25} - 0.5g = -0.5\ddot{x}$ (1)

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Question 8 continued

$\Rightarrow \quad 5 + 20x - 5 = -0.5\ddot{x}$

$\Rightarrow \quad -40x = \ddot{x}$ 1

$\Rightarrow \quad w^2 = 40 \quad (SHM)$

$\Rightarrow \quad w = \sqrt{40}$

Period $= \frac{2\pi}{w}$

$\Rightarrow T = \frac{2\pi}{\sqrt{40}} = \frac{2\pi}{2\sqrt{10}} = \frac{\pi}{\sqrt{10}}$ 1 1

b) Recall that $v^2 = w^2(A^2 - x^2)$, $E_k = \frac{1}{2}mv^2$

$2.5 = \frac{1}{2}(0.5)v^2 \Rightarrow v^2 = 10$ 1

The maximum speed is when $x = 0$.

$10 = 40(a^2 - 0)$

$\Rightarrow \quad A = 0.5$ 1

$AB = l + e + A = 1.25 + 0.25 + 0.5 = 2m.$ 1

c) The string will go slack when $x = -0.25$.

Recall that $x = A\cos wt$.

$-0.25 = 0.5\cos(\sqrt{40}\, t_1)$ 1

$\Rightarrow t_1 = 0.33$ 1

P 7 2 1 1 4 A 0 2 9 3 2

Question 8 continued

After the string has gone slack, we need to find the initial speed.

$U^2 = w^2(A^2 - x^2)$

$\Rightarrow U^2 = 40(0.5^2 - 0.25^2)$ 1 $= \sqrt{7.5}$

The displacment is 0 and acceleration is $-g$

$S = ut + \frac{1}{2}at^2$

$0 = \sqrt{7.5}\, t_2 - \frac{1}{2} \times 10 \times t_2^2$

$\Rightarrow t_2 = 0.547$

As the particle comes back down, t_1 is repeated.

$t = 2t_1 + t_2$ 1 $= 1.2s$ 1